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Is There Really A Green Paradox?

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IS THERE REALLY A GREEN PARADOX?¹

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Abstract

The Green Paradox states that, in the absence of a tax on CO₂ emissions, subsidizing a renewable backstop such as solar or wind energy brings forward the date at which fossil fuels become exhausted and consequently global warming is aggravated. We shed light on this issue by solving a model of depletion of non-renewable fossil fuels followed by a switch to a renewable backstop, paying attention to timing of the switch and the amount of fossil fuels remaining unexploited. We show that the Green Paradox occurs for relatively expensive but clean backstops (such as solar or wind), but does not occur if the backstop is sufficiently cheap relative to marginal global warming damages (e.g., nuclear energy) as then it is attractive to leave fossil fuels unexploited and thus limit CO₂ emissions. We show that, without a CO₂ tax, subsidizing the backstop might enhance welfare. If the backstop is relatively dirty and cheap (e.g., coal), there might be a period with simultaneous use of the non-renewable and renewable fuels. If the backstop is very dirty compared to oil or gas (e.g., tar sands), there is no simultaneous use. The optimum policy requires an initially rising CO₂ tax followed by a gradually declining CO₂ tax once the dirty backstop has been introduced. We also discuss the potential for limit pricing when the non-renewable resource is owned by a monopolist.

Keywords: Green Paradox, Hotelling rule, non-renewable resource, renewable backstop, global warming, carbon tax, limit pricing

JEL codes: Q30, Q42, Q54

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1. Introduction

The accumulation of CO₂ due to the extraction and use of fossil fuels is the main cause of climate change. Consequently, “.. the ‘pure’ mining problem must be coupled with the ‘pure’ pollution problem and questions like these become relevant: which should we run out first, air to breathe or fossil fuels to pollute the air we breathe?” (d’Arge and Kogiku, 1973). In the design of optimal climate policy one could neglect the exhaustibility of fossil fuels by arguing that they are abundant until the far future, as is the case for coal or oil from tar sands. However, this may lead to the failure of climate policy. In the absence of renewable resources such as solar or wind energy, some fossil fuels such as oil and gas are essentially available in limited amounts and their optimal intertemporal use needs to be determined in conjunction with any adverse effects this may have on global warming. The optimal policy of extracting such fossil fuels and combating climate change should take into account the order in which the fuels are to be extracted. In doing so, differences in extraction costs for the various sources of energy as well as differences in the contributions the resources make to climate change play a role. With the availability of renewable backstops these problems persist. In addition, the timing, order and speed of extraction in conjunction with the introduction of the backstop are crucial for future welfare. Our aim is to present a dynamic welfare analysis in a world where climate change poses a serious negative externality. We explicitly consider exhaustibility of some fossil fuels, but also look at backstops⁴. Backstops are defined as renewable resources that are perfect substitutes for fossil fuel and not constrained by exhaustibility. Our special interest in the role of backstops is motivated by the argument that subsidizing backstops may have negative detrimental climate effects (Sinn, 2008). Sinn’s ‘Green Paradox’ has received a lot of attention in the press, but recently also has been scrutinized more rigorously (e.g., Hoel, 2008; Gerlagh, 2009). It is our aim to critically review this argument in a model that is more complex and more comprehensive than the one employed by Sinn. Our extensions concern the following aspects.

In the first place, we study the situation where marginal extraction costs of the non-renewable resource depend on the existing stock, whereas Sinn only considers constant marginal extraction costs. It follows that lowering the cost of supplying the renewable backstop may lead to a positive remaining stock of the fossil fuels in case the backstop price is lower than the marginal extraction costs at low resource stocks.

⁴ Papers addressing externalities and exhaustibility, but abstracting from a backstop, include Krautkraemer (1985), who is mainly interested in preservation in view of amenity values, Withagen (1994), who shows that initial use of the exhaustible resource is smaller than without the externality, and Sinclair (1992) and Ulph and Ulph (1994), who deal with optimal (dynamic) taxation of fossil fuels in view of their detrimental effect on the environment.

Secondly, we make a distinction between backstops according to their production costs as well as according to the degree they contribute to the atmospheric concentration of CO₂. Table 1 below gives a coarse taxonomy of the various types of backstop.

Table 1: Alternative energy sources to conventional oil and gas

Backstop	Expensive: $b > G(0)$	Cheap: $b < G(0)$
Zero CO ₂ emissions: $\psi = 0$	Solar/wind/advanced nuclear: Green Paradox	Nuclear: No Green Paradox
Cleaner: $\psi < 1$	CCS coal	-
Bit dirty: ψ bit bigger than 1	-	Coal: ψ is 1.3 and 1.8 relative to oil and gas, respectively
Very dirty: ψ much bigger than 1	Tar sands: ψ is 1.5 to 5	-

In this table, b denotes the unit cost of the backstop energy source and $G(S)$ with $G' < 0$ is the unit cost of extracting non-renewable oil or gas, where S denotes the stock of oil and gas reserves.⁵ We call a backstop expensive if its production cost b is larger than the maximal extraction cost of conventional oil or gas, $G(0)$. The backstop is cheap otherwise. The parameter ψ indicates how dirty the backstop is compared with conventional oil or gas. So, we normalise the emission ratio of the fossil fuel to unity. If the backstop has a ψ smaller than unity, the backstop is cleaner in terms of CO₂ emissions than conventional oil or gas. If the backstop has $\psi = 0$, it is clean. A backstop is dirty if its contribution to emissions is larger than that of conventional oil or gas, i.e., $\psi > 1$.

Thirdly, policy recommendations follow straightforwardly from the dynamic optimization results. Optimal taxes correspond to the shadow prices that are generated in the social optimum. We will characterize them, also in a dynamic setting. However, it is well known that for sure on a world scale these optimal taxes are difficult to implement for many reasons. It is well accepted that in the long run new technologies are indispensable and one could therefore advocate subsidizing the development of clean backstop technologies. From a global social welfare perspective a decrease in the cost of supplying the backstop may be beneficial, albeit not for green welfare. However, unless the reduction is realized in a costless way, the policy will in general not be first best. Following Sinn (2008), we analyze what happens if a Hotelling ramp for taxes on CO₂ emissions is politically infeasible. If the government then resorts to subsidizing solar or wind energy, as is done on a large scale in Germany, depletion of oil may occur more

⁵ Useful studies on the costs of producing various renewable and non-renewable forms of energy are Shihab-Eldin (2002), European Commission (2003), Neuhoﬀ (2004) and Paltsev et al. (2009).

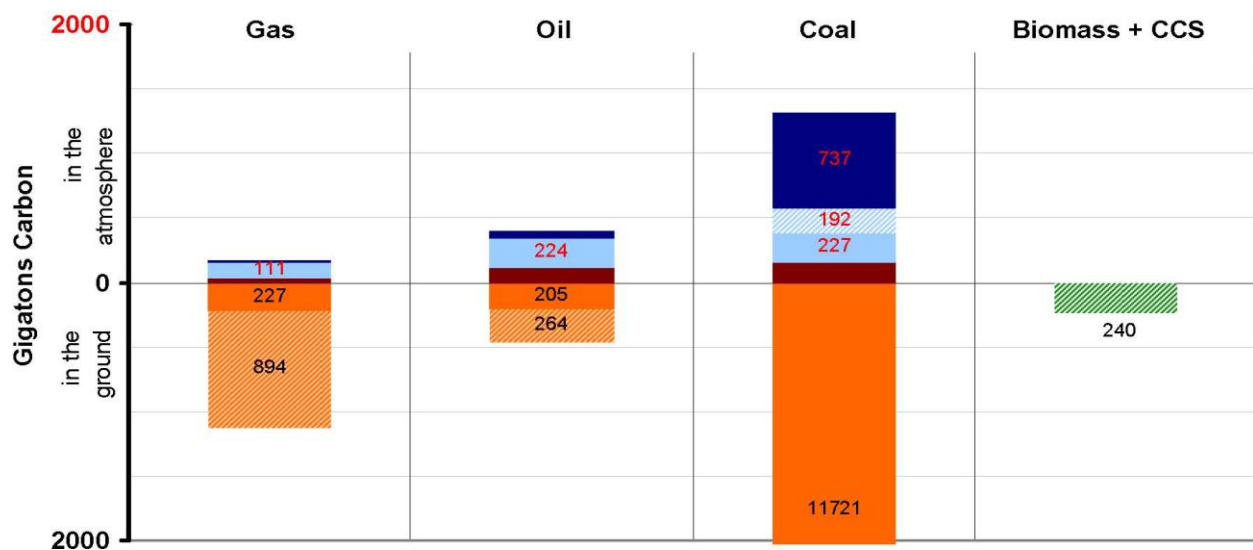
rapidly and discounting then implies that climate change damages increase. This is what Sinn has in mind when he refers to the Green Paradox. We also show that, if the atmosphere has already been polluted with a lot of CO₂ emissions, it is socially optimal to postpone depletion of oil and gas to combat global warming. We discuss the following cases:

- a. Clean and expensive backstops such as solar and wind energy. We suppose that these resources make no contribution to the accumulation of greenhouse gases. Hence, ignoring that a lot of energy might be needed to produce these technologies, we set $\psi = 0$. We also assume that these backstops are expensive, possibly not when it comes to the marginal production costs once capacity is installed, but surely it is expensive to increase capacity, there are costs to do with intermittence and especially offshore wind mills are very costly to repair. Wind energy is estimated to be at least three times as expensive as 'grey' electricity (Wikipedia). However, a recent study suggests that, as far as the electricity industry is concerned, the costs of renewable sources of energy have fallen quite a bit: solar energy is currently 50% more expensive than conventional electricity, wind energy has the same cost and is (apart from the problem of intermittence) competitive, and biomass, CCS coal/gas and advanced natural gas combined cycle have mark-ups of 10%, 60% and 20%, respectively (Paltsev et al., 2009). These mark-ups for renewable energy sources are measured from a very low base and may not be so impressive when they account for a much larger market share. We will show that the Green Paradox prevails if the backstop is becoming cheaper provided that the backstop remains expensive.
- b. At the other end of our spectrum of energy sources that do not emit CO₂, we place nuclear energy, which is deemed to be rather competitive already, possibly due to the neglect of the cost to be incurred after the plants become obsolete. We show that with this cost configuration the Green Paradox no longer holds. In as far as advanced nuclear is much more expensive than conventional energy as suggested by a mark-up of 70% (Paltsev et al., 2009), our arguments suggest that the Green Paradox may not hold.
- c. The third category consists of carbon sequestration of electricity-generating industries, which is more expensive than using conventional oil or gas but has lower CO₂ emissions.
- d. The fourth category consists of heavily polluting and expensive backstops. In this paper we treat the tar sands as such, because their reserves are much larger than the existing conventional oil and gas reserves. Although burning oil from tar sands yields same emissions as burning conventional oil, a lot of energy is used in producing oil from tar sands and therefore CO₂ emissions are at least 50% higher and in some cases perhaps even 3 to 5 times higher than those of conventional

oil. They also adversely affect the livelihood of indigenous communities via large-scale leakage of toxic waste in groundwater and destruction of ancient forests larger than the size of England. We show that in this case oil and gas reserves will be fully exhausted and that it is optimal to combine a CO₂ tax ramp with a high albeit declining CO₂ tax once the tar sands are in operation.

- e. Finally, we consider coal which is heavily polluting (electricity from coal-fired plants are 30% higher than oil-fired plants), but cheap to exploit (depending on location and soil characteristics). Also, the process of making coal liquid so that it can be a substitute for oil in transportation takes a lot of energy. We consider coal also as a backstop, since to all intents and purposes the supply of coal lasts indefinitely as may be seen from figure 1. We show that with a relatively cheap and moderately backstop as coal, it can be optimal to have a period of simultaneous use of oil or gas and coal before the switch to coal is made completely. Coal should also be taxed.

Figure 1: Coal Reserves dominate Gas and Oil Reserves



Key: The CO₂ that has been emitted and will in the future be emitted under the scenario that CO₂-equivalent levels are stabilized at 400 parts per million compatible with the objective of limiting the increase in global temperature to a maximum of 2 degrees centigrade over pre-industrial levels is: estimated energy consumption since 1990 has already led to 227, 224 and 111 Giga tons of carbon for coal, oil and gas, respectively; estimated additional consumption to hit targets 737 Gigaton for coal, but almost none for gas and oil and coal plus CCS much less, namely 192 Giga ton of carbon. The remaining bit immediately above the line is Giga tons of carbon emitted in the atmosphere due to cumulative historical energy consumption. Biomass and CCS has and will not lead to CO₂ emissions.

Under-the-ground reserves imply the following further future CO₂ emissions: probable and proven resources and reserves for gas, respectively, 894 and 227 Giga ton; oil 264 and 205 Giga ton of carbon; reserves for coal 11,721 Giga ton of carbon; biomass plus carbon, capture and storage (CCS) 240 Giga ton of carbon unless it is fully sequestered in which case there will be zero CO₂ emissions.

Source: Edenhofer and Kalkuhl (2009)

Fourthly, as has been argued that the Green Paradox is the result of rational speculative behaviour of resource-owning monopolists (Sinn, 2008; Gerlach, 2009), we analyze the implications of imperfect competition more formally. When it comes to the theory of non-renewable resources, this is the case most studied elsewhere in the literature. The reason is that neither the oil market dominated by OPEC nor the gas market dominated by Russia, Iran, Qatar and Venezuela can be characterized as competitive. It goes beyond the scope of the present paper to extend the cartel-fringe model (e.g., Groot et al., 2003) by allowing for a renewable backstop and the interactions with climate change. However, we do pay attention to the case of a resource-owning monopolist, in order to incorporate the phenomenon of limit pricing (cf., Hoel, 1978). We show that depletion under a monopolist will be slower. Moreover, if the backstop is relatively cheap, a Green Paradox need not arise.

The outline of the paper is as follows. Section 2 analyzes the socially optimal transition from conventional oil and gas to a clean renewable backstop, in the face of climate externalities and stock-dependent extraction costs. Damage from CO₂ emissions can be modelled through a negative externality in production (cf., Heal, 1985; Sinn, 2008), but we follow the mainstream approach where damage adversely affects social welfare. We abstract from capital accumulation.⁶ Section 3 studies the outcome in a decentralized market economy and shows how the social optimum can be sustained with a rising CO₂ tax. It also shows that in the second-best situation where a rising CO₂ tax is infeasible, subsidizing the backstop need not lead to a Green Paradox if this encourages private resource owners to leave more oil and gas in the soil and to switch more quickly to the clean backstop. Section 4 investigates the implications of dirty backstops such as coal or the tar sands for the extraction of oil and gas and for climate change policy. Section 5 offers some ideas on the Green Paradox and imperfect competition. Section 6 concludes and discusses policy implications.

2. Switching from dirty fossil fuels to a clean backstop: social optimum

We study optimal extraction of a scarce exhaustible, non-renewable resource (i.e., gas or oil) with a backstop kicking in when oil prices become high enough. The backstop is a perfect substitute for the non-renewable resource and its supply is infinitely elastic. To assess the Sinn (2008) arguments properly, we add climate change externalities to social welfare. The easiest way is to add a convex function in past

⁶ Golosov, et al. (2009) also study a general equilibrium model of fossil taxes and a backstop fuel, but focus on capital accumulation and ignore exhaustibility of fossil reserves. They show that optimal ad-valorem taxes on oil consumption decline over time. In line with section 5 of our paper and abstracting from a competitive fringe, Hassler et al. (2009) conclude that an oil monopoly is good for the environment.

CO2 emissions to the felicity function to capture the damage done by the CO2 emissions into the atmosphere from burning oil. A widely used description of the accumulation of CO2 in the atmosphere reads $\dot{E}(t) = q(t) - \nu E(t)$. However, in order to get tractable solutions we set $\nu = 0$.⁷ With quasi-linear preferences, the social planner's problem then reads:

$$(1) \quad \text{Max} \int_0^{\infty} \exp(-\rho t) [U(q(t) + x(t)) - G(S(t))q(t) - bx(t) - D(E(t))] dt$$

subject to $\dot{E}(t) = q(t)$, $E(0) = E_0$, the non-negativity condition $x(t) \geq 0$ and the depletion equation

$$(2) \quad \dot{S}(t) = -q(t), \quad q(t) \geq 0, \quad S(t) \geq 0, \quad S(0) = S_0, \text{ given,}$$

where ρ is the constant rate of time preference, q denotes the extraction rate, G the per unit extraction costs, x the rate of use of the backstop and b the unit cost of supplying the backstop energy source. The instantaneous utility function is U with $U' > 0$ and we suppose that at the optimum $U'' < 0$. The per unit extraction costs of the non-renewable resource are a decreasing function of the in-situ stock, $G' < 0$. Hence, as reserves diminish, unit cost of extraction rises. The set of constraints (2) implies that total current and future depletion cannot exceed reserves, $\int_0^{\infty} q(t) dt \leq S_0$. We have $E(t) = E_0 + S_0 - S(t)$ with E_0 indicating the CO2 emissions that have taken place up to time zero.

The current-value Hamiltonian function for the social planner is defined by:

$$(3) \quad H(q, x, S, \lambda, \mu) \equiv U(q + x) - G(S)q - bx - D(E) - \lambda q - \mu q,$$

where λ is the shadow price of the non-renewable resource and $\mu (> 0)$ is the shadow cost of the pollution stock. The necessary conditions for a social optimum are:

$$(3a) \quad \dot{\lambda}(t) = \rho \lambda(t) + G'(S(t))q(t), \quad \dot{\mu}(t) = \rho \mu(t) - D'(E(t)),$$

$$(3b) \quad \lim_{t \rightarrow \infty} \exp(-\rho t) [\lambda(t)S(t) - \mu(t)E(t)] = 0$$

$$(3c) \quad U'(q(t) + x(t)) - \lambda(t) - \mu(t) - G(S(t)) \leq 0, \quad q(t) \geq 0, \quad \text{c.s.,}$$

⁷ The model of this section strongly resembles the model analysed by Hoel and Kverndokk (1996). However, a major difference is that they do not perform sensitivity analysis with respect to the cost of supplying the backstop. Moreover, we assume that there is no natural decay. This implies that we can abstract from the possibility of simultaneous use of the backstop and the fossil fuel.

$$(3d) \quad U'(q(t) + x(t)) - b \leq 0, x(t) \geq 0, \text{ c.s.},$$

where c.s. refers to complimentary slackness. Define $\eta \equiv \lambda + \mu (> 0)$ as the social value of the non-renewable resources, consisting of the value of the stock, as a stock of resources, and the environmental value of keeping the stock in the ground, the necessary conditions can be interpreted as follows. The modified Hotelling rule implied by (3a) says that the rate of increase in the scarcity rent of the non-renewable resource ($\dot{\eta} / \eta$) must equal the rate of time preference ρ plus the sensitivity of the marginal cost of extraction to the stock of remaining reserves $G'(S)q$ minus marginal global warming damages, both normalized by the social value. Global warming damages imply that it is socially optimal to deplete the stock of oil and gas reserves more slowly. A rapidly increasing unit cost of extraction also slows down the depletion of oil and gas. The decentralized market economy does not internalize the damages done by CO2 emissions and therefore leads to too rapid extraction of oil and gas reserves (see section 3). The transversality condition (3b) states that the present values of the social value of the remaining stock of oil and gas and of the social cost of the pollution stock vanish as time goes to infinity. Equation (3c) says that no resource extraction takes place if the marginal utility of the resource is below marginal extraction costs plus the social cost of the resource. Equation (3d) says that the backstop is used unless the marginal utility of energy falls short of the supply price b . In order to have an interesting problem, we assume that, for the initial stock of non-renewable reserves, the marginal extraction cost is smaller than the maximum price of fuel, i.e., $G(S_0) < U'(0)$. Furthermore, we suppose that the maximum price of fuel is such that use of the backstop energy source is profitable, $U'(0) > b$.

From (3c), we have $q(t) + x(t) \geq \bar{x}$ where $U'(\bar{x}) = b$ gives $\bar{x} \equiv F(b)$, $F' = 1/U'' < 0$. Therefore, as long as there is resource extraction, the pollution stock increases and the per unit extraction costs increase as well. Moreover, since we abstract from natural decay of the CO2 concentration in the atmosphere, resource extraction will come to an end, say at T . After T there will never again be resource extraction. At any instant of time $t < T$, $q(t)$ and $x(t)$ cannot both be positive.⁸ We thus have

$$(4) \quad x(t) = F(b), q(t) = 0, S(t) = S(T), E(t) = E(T) \text{ and } U(q(t) + x(t)) = b, \quad \forall t \geq T,$$

$$(5) \quad \lambda(T)S(T) = (\mu(T) - D'(E(T)) / \rho)E(T),$$

⁸ In case of simultaneous use equations (3c) and (3d) give $U'(q(t) + x(t)) = b = \eta(t) + G(S(t))$, so (3a) implies $\dot{\eta}(t) = -G'(S(t))\dot{S}(t) = \rho[b - G(S(t))] + G'(S(t))q(t) - D'(E_0 + S_0 - S(t))$ or $\eta(t) = D'(E_0 + S_0 - S(t)) / \rho > 0$ and thus $\dot{\eta}(t) = D''(E(t))q(t) / \rho > 0$. But we also have $\dot{\eta}(t) = G'(S(t))q(t) < 0$. Both cannot be true at the same time, so $q(t)$ and $x(t)$ cannot both be strictly positive in the interval up to time T .

where the latter is the transversality condition⁹.

2.1. How much oil and gas to leave in the soil and when to switch to the backstop?

Two possibilities arise. If no oil/gas is left in situ it follows from (5) that $\mu(T) = D'(E_0 + S_0) / \rho$. Moreover, from (3c) and (3d), $\lambda(T) = b - G(0) - \mu(T)$. If not all oil/gas is depleted, its value in situ must be zero, $\lambda(T) = 0$. We thus have the requirement that the present value of marginal future global warming damages of one unit more of remaining oil reserves (*FMD*) equals the marginal benefit of extracting an extra unit of oil (*MB*) at the time of the switch to the backstop:

$$(6) \quad FMD = D'(E_0 + S_0 - S(T)) / \rho = b - G(S(T)) \equiv MB,$$

which is strictly positive to reflect that keeping more oil/gas in situ implies less CO2 emissions and thus less global warming damages. This yields the stock of remaining oil/gas reserves at the time of the switch, $S(T) = S_T$. The solution $S(T)$ may be negative if $b > G(0)$ and the aversion to global warming is small (i.e., preferences for a low CO2 stock are low) in which a case the resource will be fully exhausted at the moment the backstop energy source takes over and $S(T) = 0$.

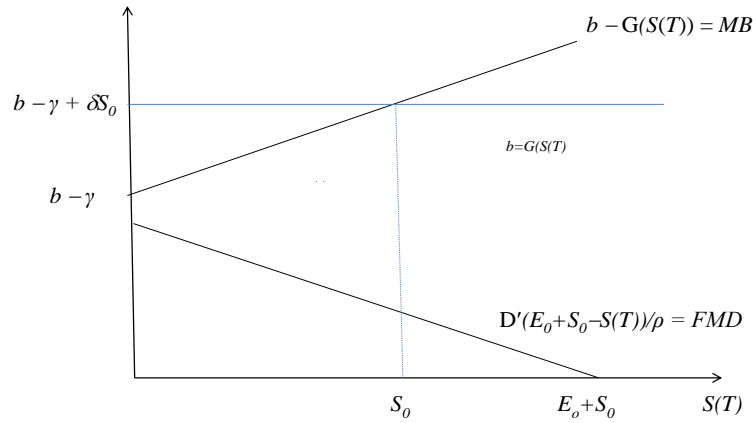
Figure 2 illustrates how (6) strikes the optimal balance between, on the one hand, more conventional oil and gas extraction for immediate benefit, and, on the other hand, less extraction to limit future global warming. The figures are drawn for the specifications $U(q) = \alpha q - \beta q^2 / 2$, $G(S) = \gamma - \delta S$ and $D(E) = \frac{1}{2} \kappa E^2$, $\kappa > 0$, which will be used throughout by way of illustrative example, but the conclusions apply to the general model. We suppose that it is attractive to use oil and gas given the cost of extracting it and the discounted value of marginal global warming damages (i.e., $\alpha > \gamma + \kappa(E_0 + S_0) / \rho$). If marginal global warming damages are not very important (a low κ , a low value of $S_0 + E_0$) and/or a high value of the rate of discount ρ and costs of supplying oil and gas are low relative to the cost of supplying the backstop (low γ , high b), the *FMD* locus will hit the vertical axis before it intersects the *MB* locus. In that case, it is optimal to fully extract conventional oil/gas reserves before switching to the backstop energy source as in panel (a) of figure 2. However, if marginal global warming damages are believed to

⁹ To see this, observe that $\forall t > T$, $b \leq \eta(t) + G(S(T))$, $\dot{\lambda} / \lambda = \rho$ and $\dot{\mu} = \rho\mu - D'(E(T))$. The solution to the latter is $\mu(t) = D'(E(T)) / \rho + [\mu(T) - D'(E(T)) / \rho] \exp(\rho(t - T))$. The transversality condition thus reads $\lambda(T)S(T) \exp(-\rho T) - [\mu(T) - D'(E(T)) / \rho] E(T) \exp(-\rho T) = 0$.

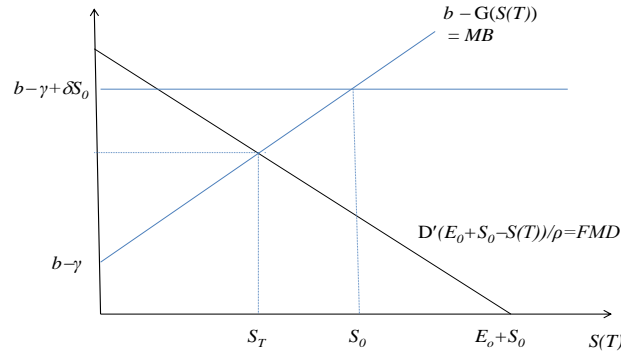
be important as is argued by the Stern Review (2007) which adopts a very low discount rate, the *FMD* locus will cross the *MB* locus as in panel (b) of figure 2, so that it is optimal to leave oil/ gas reserves in the soil ($S_0 < S(T) = S_T > 0$). If the backstop is really cheap, the climate challenge is very acute, and not much oil/gas is left in situ ($b < \gamma - \delta S_0 + \kappa E_0 / \rho$), it is optimal to never use oil/gas and start using the backstop straight away (the intersection point then lies to the right of S_0).

Figure 2: Marginal global warming damages and marginal benefits of oil extraction

(a) Full exhaustion: expensive backstop and modest climate challenge ($b > \gamma + \kappa(E_0 + S_0) / \rho$)



(b) Partial exhaustion: cheap backstop and acute climate challenge ($b < \gamma + \kappa(E_0 + S_0) / \rho$)



We see from (6) that a lower cost of supplying the backstop (lower b) shifts down the MB schedule and leads to a higher remaining stock of conventional oil and gas reserves at the time of the switch to the backstop. If the problem of global warming is already acute and the initial stock of oil and gas reserves is high, it is also optimal to leave a larger share of oil and gas reserves in situ. More patience as advocated by the Stern Review (i.e., low value of ρ) leads to more concern with global warming damages in the distant future, hence a bigger stock of conventional oil and gas reserves is left in situ.

From the first-order conditions (3a)-(3d), we get for $t \in [0, T]$ the following differential equation:

$$(7) \quad \rho \left[U'(-\dot{S}(t)) - G(S(t)) \right] + U''(-\dot{S}(t))\ddot{S}(t) = D'(E_0 + S_0 - S(t)).$$

With the functional forms introduced previously, we obtain the following proposition.

Proposition 1: With global warming externalities, the socially optimal outcome gives rise to slower depletion of the stock of the non-renewable resource. The stocks of oil and gas are given by:

$$(8) \quad S(t) = \left[\frac{(S(T) + \Gamma) \exp(-s_1 T) - (S_0 + \Gamma) \exp((s_2 - s_1)T)}{1 - \exp[(s_2 - s_1)T]} \right] \exp(s_1 t) \\ + \left[\frac{S_0 + \Gamma - (S(T) + \Gamma) \exp(-s_1 T)}{1 - \exp[(s_2 - s_1)T]} \right] \exp(s_2 t) - \Gamma, \quad t \leq T,$$

where $s_1 = \frac{1}{2} \rho + \frac{1}{2} \sqrt{\rho^2 + 4(\rho\delta + \kappa) / \beta} > \rho$, $s_2 = \frac{1}{2} \rho - \frac{1}{2} \sqrt{\rho^2 + 4(\rho\delta + \kappa) / \beta} < 0$ and

$\Gamma \equiv \frac{\rho(\alpha - \gamma) - \kappa(E_0 + S_0)}{\rho\delta + \kappa} > 0$. The time of the switch from oil and gas to the backstop is given by:

$$(9) \quad T = T \left(\overset{+}{S}_0, 0, \overset{+}{b}, \overset{-}{\alpha}, \overset{+}{\gamma}, \delta, \rho, \beta, \overset{+}{E}_0, \kappa \right) \text{ if } b \geq \gamma + \kappa(E_0 + S_0) / \rho \\ T = T \left(\overset{+}{S}_0, \frac{\rho(\gamma - b) + \kappa(E_0 + S_0)}{\kappa + \rho\delta}, \overset{-}{b}, \overset{-}{\alpha}, \overset{+}{\gamma}, \delta, \rho, \beta, \overset{+}{E}_0, \kappa \right) \text{ if } b < \gamma + \kappa(E_0 + S_0) / \rho,$$

where the pluses and minuses in expression (9) indicate signs of the partial derivatives of $T(\cdot)$. The stock of remaining non-renewable reserves at the time of the switch is given by:

$$(10) \quad S(T) = 0 \text{ if } b \geq \gamma + \frac{\kappa(E_0 + S_0)}{\rho} \text{ and } S(T) = \frac{\rho(\gamma - b) + \kappa(E_0 + S_0)}{\kappa + \rho\delta} \equiv S_T \text{ otherwise.}$$

Afterwards, use of the backstop is given by $x(t) = (\alpha - b)/\beta > 0$, $\forall t \geq T$.

Proof: Equation (7) yields $\beta \ddot{S} - \rho \beta \dot{S} - (\rho \delta + \kappa)S = \rho(\alpha - \gamma) - \kappa(E_0 + S_0)$. The solution for $t \leq T$ is thus given by $S(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t) - \Gamma$. The characteristic equation $\beta s^2 - \rho \beta s - (\rho \delta + \kappa) = 0$ give the roots s_1 and s_2 . Using the boundary conditions $K_1 + K_2 - \Gamma = S_0$ and $K_1 \exp(s_1 T) + K_2 \exp(s_2 T) - \Gamma = S(T)$, we solve for K_1 and K_2 and obtain (7). Continuity of the Hamiltonian, $q(T) = -\dot{S}(T) = -s_1 K_1 \exp(s_1 T) - s_2 K_2 \exp(s_2 T) = F(b)$, gives:

$$(11) \quad F(b) = \frac{\alpha - b}{\beta} = \frac{(S_0 + \Gamma)(s_1 - s_2) \exp(s_2 T) + (S(T) + \Gamma)[s_2 \exp((s_2 - s_1)T) - s_1]}{1 - \exp((s_2 - s_1)T)} \\ \equiv \Phi \left(\bar{T}, \bar{S}_0, \bar{S}(T), \bar{\Gamma}, s_1, s_2 \right).$$

With $s_1 > \rho > 0$ and $s_2 < 0$, the denominator on the right-hand side is strictly positive and increasing in T ; given that we suppose that $\Gamma > 0$, the numerator is decreasing in T ,

$$\frac{\partial \text{Num}}{\partial T} = \underbrace{s_2(s_2 - s_1) \exp(s_2 T)}_{+} \left[\underbrace{(S(T) \exp(-s_1 T) - S_0)}_{-} - \underbrace{\Gamma[1 - \exp(-s_1 T)]}_{+} \right] < 0. \text{ Hence, } \Phi_T < 0. \text{ The signs of the}$$

derivatives of $\Phi(\cdot)$ with respect to S_0 and $S(T)$ follow immediately. The derivative with respect to Γ is

$$\frac{(s_1 - s_2) \exp(s_2 T) + s_2 \exp((s_2 - s_1)T) - s_1}{1 - \exp((s_2 - s_1)T)}, \text{ which tends to } -s_1 < 0 \text{ as } T \rightarrow \infty. \text{ Since the numerator decreases and the}$$

denominator increases as T increases, the derivative of $\Phi(\cdot)$ with respect to Γ must be negative. Making use of the dependence of s_1 and s_2 on ρ , δ , β and κ and of the definition of Γ , we use (11) to solve for $T(\cdot)$ as in (9). Since s_1 and s_2 do not depend on S_0 , S_T , b , α and γ , the signs of the partial derivatives of $T(\cdot)$ with respect to S_0 , S_T , b , α and γ are unambiguous. The signs of the partial derivatives with respect to ρ , δ , β and κ are ambiguous. (10) follows from the terminal condition (6), $\eta(T) + G(S(T)) = \kappa[E_0 + S_0 - S(T)]/\rho + [\gamma - \delta S(T)] = b$ provided that $S(T) \geq 0$. Together with the function $T(\cdot)$, equation (10) gives (9). Q.E.D.

We thus distinguish three regimes for the social optimum:

- Full exhaustion of non-renewables if the backstop is expensive compared to initial marginal extraction costs of non-renewables (high b , low γ), the global warming challenge is not too acute (low E_0 and S_0), and the social discount rate is high (high ρ), that is $b > \gamma + \kappa(E_0 + S_0)/\rho$.
- Partial exhaustion if the backstop becomes cheap enough as extraction of gas and oil becomes more expensive, the global warming challenge is more acute, and the discount rate is low enough, that is $\gamma - \delta S_0 + \kappa E_0 / \rho < b < \gamma + \kappa(E_0 + S_0)/\rho$.
- If the backstop is always cheaper than marginal extraction costs of non-renewables, initial CO2 concentration is very high, and the social discount rate is very low, that is $b < \gamma - \delta S_0 + \kappa E_0 / \rho$, the backstop is immediately introduced and oil and gas reserves remain completely unexploited.

As technical progress reduces the cost of the backstop or a lower discount rate is used, one moves from a regime of full exhaustion to partial exhaustion and eventually zero exhaustion of oil and gas reserves.

The case that conventional oil and gas reserves are fully exhausted ($S_T = 0$) may be relevant for solar or wind energy. A higher initial CO2 concentration in the atmosphere (higher E_0) then leads to slower depletion of conventional oil and gas reserves and the time of the switch to the backstop energy source is postponed. However, if initial marginal environmental damages and the maximum cost of supplying oil or gas are high relative to the cost of the backstop, as one might expect for nuclear energy, it is optimal to not fully exhaust oil and gas reserves to avoid global warming damages. In that case, a higher initial CO2 concentration (higher E_0) implies that one wants to keep a larger stock of oil and gas in situ and therefore there is an extra force that brings forward the time of the switch to the backstop.

Even if it is optimal to fully exhaust oil and gas reserves, there are two opposing effects on the time to the switch to the backstop fuel of more rapidly rising marginal CO2 damages (higher κ), namely a positive effect via Γ and a negative effect via s_1 and s_2 . So it is a priori not clear whether this slows down depletion of non-renewables or not. If it is optimal to leave oil and gas reserves in situ, there are extra effects via S_T .

2.2. Green welfare and the cost of supplying the backstop energy source

To assess the effect of a marginal change in the cost of the backstop on green welfare, we define:

$$(12) \quad \Lambda \equiv -\int_0^\infty \frac{1}{2} \kappa E(t)^2 \exp(-\rho t) dt = \Lambda_R + \Lambda_B,$$

$$\text{where } \Lambda_R \equiv -\int_0^T \frac{1}{2} \kappa [E_0 + S_0 - S(t)]^2 \exp(-\rho t) dt \text{ and } \Lambda_B \equiv -\left(\frac{\kappa}{2\rho}\right) [E_0 + S_0 - S(T)]^2 \exp(-\rho T).$$

If it is optimal to fully exhaust oil/gas reserves, a lower cost of the backstop (lower b) leads to a more rapid switch to the backstop (lower T). Due to discounting and convexity of global warming damages, one may argue that faster depletion of oil/gas reserves must lead to worsening of green welfare. However, this conjecture is not valid if the backstop is relatively cheap compared with marginal extraction costs of oil and gas and the present value of marginal global warming damages.

Proposition 2: If $b > \gamma + \kappa(E_0 + S_0) / \rho$, a cheaper backstop reduces green welfare:

$$(13a) \quad \frac{\partial \Lambda}{\partial b} = \int_0^T \kappa E(t) \frac{\partial S(t)}{\partial b} \exp(-\rho t) dt > 0$$

If $b < \gamma + \kappa(E_0 + S_0) / \rho$, a cheaper backstop increases green welfare:

$$(13b) \quad \frac{\partial \Lambda}{\partial b} = - \underbrace{\left(\frac{\kappa}{\kappa + \rho \delta} \right) (E_0 + S_0 - S_T) \exp(-\rho T)}_{-} + \underbrace{\int_0^T \kappa E(t) \frac{\partial S(t)}{\partial b} \exp(-\rho t) dt}_{-} < 0.$$

A cheaper backstop always leads to higher total welfare.

Proof: We have $\frac{\partial \Lambda}{\partial b} = \frac{1}{2} \kappa [E_0 + S_0 - S(T)] \exp(-\rho T) \frac{\partial T}{\partial b} + \frac{\kappa}{\rho} [E_0 + S_0 - S(T)] \exp(-\rho T) \frac{\partial S(T)}{\partial b}$. Using the Leibniz integration rule to differentiate Λ_R with respect to b , we obtain the expression

$$\frac{\partial \Lambda}{\partial b} = - \frac{1}{2} \kappa [E_0 + S_0 - S(T)] \exp(-\rho T) \frac{\partial T}{\partial b} + \int_0^T \kappa E(t) \frac{\partial S(t)}{\partial b} \exp(-\rho t) dt. \text{ We thus obtain}$$

$$\frac{\partial \Lambda}{\partial b} = \frac{\kappa}{\rho} [E_0 + S_0 - S(T)] \exp(-\rho T) \frac{\partial S_T}{\partial b} + \int_0^T \kappa E(t) \frac{\partial S(t)}{\partial b} \exp(-\rho t) dt. \text{ If } b \geq \gamma + \kappa(E_0 + S_0) / \rho, \text{ then according to}$$

proposition 1, $S(T) = 0$ and the first term is zero. Since a marginal reduction in b reduces T and $q(t) \geq 0, \forall t \leq T$, at any point in the interval $t \in (0, T]$ $S(t)$ is lower and consequently the second term in the expression for $\partial \Lambda / \partial b$ is positive. Hence, we have (13a) and $\partial \Lambda / \partial b < 0$. If $b < \gamma + \kappa(E_0 + S_0) / \rho$, then according to proposition 1, $S_T > 0$.

Using the expression for S_T given in (10) to evaluate the derivative with respect to b , we obtain (13b). The sign of the first term in (13b) follows immediately. The sign of the second term in (13b) is less obvious. We know that a lower b leads to a higher S_T and want to show that a lower b reduces $S(t)$ for all $t \in (0, T]$. Let us denote variables (and co-states) corresponding to the marginally lower value of b by an asterisk. We will prove that $q^*(t) < q(t), \forall t \in [0, T)$. To do so, we suppose $q^*(t) > q(t)$ for some initial period of time in the interval $[0, T^*)$ and work towards a contradiction. Since $q^*(0) > q(0)$, we have $\eta^*(0) < \eta(0)$ as both trajectories start with the same value of S_0 .

Inspecting equations (3c) and (3a) and noting that G' is a negative constant, we establish that as long as $q^*(t) > q(t)$ there will be more pollution. Hence, $\eta^*(t) < \eta(t)$ for all $t \in [0, T^*)$. From (3a) and (3c), we have

$$\dot{q}(t) = \frac{\rho \eta(t) - D'(E(t))}{U''} \text{ and thus } \dot{q}(0) - \dot{q}^*(0) = [\rho \eta(0) - \rho \eta^*(0)] / U'' < 0. \text{ It must thus be the case that } q^*(t)$$

falls less rapidly than $q(t)$ as time progresses. But $S^*(t) < S(t)$, so at some point $q^*(t) > q(t)$ in the interval $[0, T)$.

But that is not possible. Hence, $q^*(0) < q(0)$ and $q^*(t) < q(t)$ for the entire interval $t \in [0, T)$. Hence,

$S^*(t) > S(t)$ for the entire interval $t \in [0, T]$. It follows that the second term in (13b) is negative. Q.E.D.

If the backstop is relatively expensive and the present value of global warming damages is not too high, it is optimal to fully exhaust conventional oil and gas reserves. A lower cost of the backstop then brings forward the date of exhaustion and switch to the backstop. Furthermore, it curbs green welfare. However, if the backstop is cheap and the global warming challenge acute, it is not optimal to fully exhaust oil and gas reserves. We then recall from (9) that a cheaper backstop brings forward the switch to the backstop

even more, that is $T_b - \left(\frac{\rho}{\kappa + \rho\delta} \right) T_{S_T} > T_b > 0$ where T_b and T_{S_T} denote the partial derivatives of the function $T(\cdot)$ with respect to b and S_T , respectively. Furthermore, (10) indicates that it is optimal to leave more oil and gas in situ at the time of the switch. A cheaper backstop cuts extraction of oil and gas as a greater proportion of reserves are kept in situ. In that case, climate damages will be less.

3. Switching from dirty fossil fuels to a clean backstop: competitive decentralized market outcome

To assess how the social optimum can be sustained in a competitive market economy, we must consider behaviour of households and competitive resource owners. Households maximize $U(q+x)+C$ subject to the budget constraint, $C + p(q+x) = A - T$, where C , p , A and T denote consumer expenditures on all other commodities than fuel, the market price of fuel, endowment of households and lump-sum taxes, respectively. Households thus set $U'(q+x) = p$, so demand for fuel is $q+x = F(p)$. We assume that the mining company has access to the backstop. This is equivalent to having a separate mining company and another company supplying the backstop in competition with each other. Taking the price of fuel, the carbon tax and the backstop subsidy as given, the resource-owning firms maximize profits,

$\int_0^\infty \{ p(t)[q(t) + x(t)] - [G(S(t)) + \tau(t)]q(t) - (b - \sigma)x(t) \} \exp(-\rho t) dt$, subject to the depletion equation (2), where the carbon tax and the (constant) backstop subsidy are indicated by τ and σ , respectively. This yields the first-order conditions:

$$(3a') \quad \dot{\omega}(t) = \rho\omega(t) + G'(S(t))q(t),$$

$$(3b') \quad \lim_{t \rightarrow \infty} \exp(-\rho t)\omega(t)S(t) = 0,$$

$$(3c') \quad p(t) - G(S(t)) - \tau(t) - \omega(t) \leq 0, \quad q(t) \geq 0, \quad \text{c.s.},$$

$$(3d') \quad p(t) - b + \sigma \leq 0, \quad x(t) \geq 0, \quad \text{c.s.},$$

where ω is the private marginal value of non-renewables. Equation (3a') yields the Hotelling rule, which states that the rate of increase in the scarcity rent of non-renewables must equal the rate of time preference ρ plus the sensitivity of the marginal cost of extraction to the stock of remaining reserves $G'(S)q$, normalized by the shadow price of non-renewables. Comparing this with (3a), we see that the shadow price rises more quickly than in the social optimum so that depletion occurs too fast in a market economy

unless the CO2 tax corrects for this externality. Equation (3c') says that oil/gas extraction does not take place if the fuel price is below marginal extraction costs plus the CO2 tax. Equation (3d') says that the backstop is used unless the fuel price falls short of the supply price, net of the subsidy, $b - \sigma$.

It follows from (3c') that $q(t) + x(t) \geq \bar{x}$, where $\bar{x} \equiv F(b - \sigma)$ follows from $U'(\bar{x}) = b - \sigma$.

Let us assume that the market outcome also has an initial interval of time $[0, T]$ where only non-renewables are used at a decreasing rate and the backstop is not used, and afterwards only the backstop is used. Along the first interval, we thus have

$$(7') \quad -U''(-\dot{S}(t))\ddot{S}(t) - \dot{\tau} = \rho \left[U'(-\dot{S}(t)) - G(S(t)) - \tau \right], \quad 0 \leq t \leq T,$$

with boundary conditions $S(0) = S_0$ and $-\dot{S}(T) = q(T) = F(b - \sigma)$. The third boundary condition states that the backstop is introduced when the fuel price has risen to the cost of the backstop, net of the subsidy, that is at the instant where $p(T) = b - \sigma$ (else the Hamiltonian is not continuous). Since after the switch to the backstop, consumption of the backstop decreases in its cost and consumption of non-renewables is zero, $x(t) = F(b - \sigma)$, $q(t) = 0 \forall t \geq T$, we obtain discounted future welfare at the time of the switch as the present value of the utility of consuming the backstop:

$$(14) \quad V(T) \equiv \int_T^\infty \exp(-\rho(t - T)) U(x(t)) dt = U(F(b - \sigma)) / \rho.$$

If non-renewables are left in the soil, their in-situ value is zero, $\omega(T) = 0$, and the remaining stock follows from (3c') and (3d'), $b - \sigma = G(S(T)) + \tau(T)$. So, the remaining stock increases in the tax rate on non-renewables and the backstop subsidy. If no oil/gas is left in situ, $b - \sigma = G(0) + \tau(T) + \omega(T)$. Making use of (3c'), (3d') and the third boundary condition at the time of the switch, we obtain:

$$(15) \quad \begin{aligned} & b - \sigma = G(S(T)) + \tau(T), \omega(T) = 0 \text{ if } b - \sigma < G(0) + \tau(T), \\ & b - \sigma > G(S(T)) + \tau(T), \omega(T) = b - \sigma - G(0) - \tau(T) > 0, \text{ else.} \end{aligned}$$

If the marginal extraction costs of non-renewables are less than the net cost of the backstop for any level of the stock, non-renewables must be fully exhausted before the backstop takes over, so $S(T) = 0$. In that case, a backstop subsidy leads to an earlier exhaustion of non-renewables and to a faster rate of oil/gas extraction. If $G(0) + \tau(T) > b - \sigma$, the resource will not be fully exhausted and a backstop subsidy implies that more oil and gas is left in situ at the time of the switch, because the cut-off point is where $G(S(T)) + \tau(T) = b - \sigma$. The extraction rate now decreases if extraction of non-renewables takes place.

In the sequel we consider a specific tax schedule that allows us to derive the optimal time of transition to the backstop. Our chosen functional forms give a linear demand function given by $F(p) = (\alpha - p) / \beta$.

We suppose that it is attractive to introduce the backstop at some point, so assume that the cost of the backstop is less than the maximum price ($b < \alpha$).¹⁰

Proposition 3: In a decentralized market economy with an ad-hoc tax schedule given by

$\tau(t) = \tau_0 \exp(\rho t)$ the time path of the stock of oil and gas is given by (8) with $\Gamma \equiv \frac{\alpha - \gamma}{\delta} > 0$,

$s_1 = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 + 4\rho\delta/\beta} > \rho$ and $s_2 = \frac{1}{2}\rho - \frac{1}{2}\sqrt{\rho^2 + 4\rho\delta/\beta} < 0$. The time of the switch to the backstop follows from

$$(9') \quad T = T^* \left(S_0^+, b - \sigma, \tau_0, \alpha, \gamma, \delta, \rho, \beta \right).$$

where $T_{\tau_0}^* = 0$ and $T_{\alpha}^* < 0$ in case $S(T) = 0$ and $T_{\tau_0}^* < 0$ in case $S(T) > 0$. The stock of remaining non-renewable reserves at the time of the switch is given by:

$$(10') \quad S(T) = 0 \text{ if } b - \sigma \geq \gamma + \tau_0 \exp(\rho T) \text{ or } S(T) = [\gamma - b + \sigma + \tau_0 \exp(\rho T)] / \delta \equiv S_T \text{ else.}$$

Afterwards, use of the backstop is given by $x(t) = (\alpha - b + \sigma) / \beta > 0$, $\forall t \geq T$.

Proof: Equation (7) becomes $\beta \ddot{S}(t) - \rho \beta \dot{S}(t) - \rho \delta S(t) = \rho(\alpha - \gamma - \tau) + \dot{\tau}$. The solution trajectory $S(t)$ for $t \leq T$ is derived in similar fashion as in proposition 1. Hence, $S(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t) - (\alpha - \gamma) / \delta$ with

$$S_0 = K_1 + K_2 - (\alpha - \gamma) / \delta, \quad S(T) = K_1 \exp(s_1 T) + K_2 \exp(s_2 T) - (\alpha - \gamma) / \delta \text{ and}$$

$$(\alpha - b + \sigma) / \beta = -s_1 K_1 \exp(s_1 T) - s_2 K_2 \exp(s_2 T). \text{ We need a fourth equation to jointly determine}$$

K_1, K_2, T and $S(T)$. To that end we consider $b - \sigma = \gamma - \delta S(T) + \tau_0 \exp(\rho T)$. If the solution of the four equations system has $S(T) > 0$ then indeed the stock of non-renewables should not be depleted. If the solution has $S(T) = 0$, full exhaustion takes place. Using the first three equations we get, as in proposition 1,

$$(11') \quad F(b - \sigma) = \frac{\alpha - b + \sigma}{\beta} = \frac{(S_0 + \Gamma)(s_1 - s_2) \exp(s_2 T) + (S(T) + \Gamma)[s_2 \exp((s_2 - s_1)T) - s_1]}{1 - \exp((s_2 - s_1)T)} = \Phi \left(T, S_0^+, S(T)^-, \Gamma, s_1, s_2 \right).$$

Assuming we have $S(T) = [\gamma - b + \sigma + \tau_0 \exp(\rho T)] / \delta > 0$ and substituting $\Gamma \equiv (\alpha - \gamma) / \delta$, equation (11') can be rewritten as

¹⁰ We also suppose that it is interesting to extract non-renewables, that is we suppose that the initial price exceeds the initial marginal cost of oil/gas extraction ($\alpha - \beta q(0) > \gamma - \delta S_0 > 0$).

$$\frac{\alpha - b + \sigma}{\beta} = \frac{\left(S_0 + \frac{\alpha - \gamma}{\delta}\right)(s_1 - s_2) \exp(s_2 T) + \left(\frac{\gamma - b + \sigma + \tau_0 \exp(\rho T)}{\delta} + \frac{\alpha - \gamma}{\delta}\right)[s_2 \exp((s_2 - s_1)T) - s_1]}{1 - \exp((s_2 - s_1)T)}.$$

The denominator of the right-hand side is strictly positive and decreasing in T . The derivative of the numerator of the right-hand side with respect to T reads

$$s_2(s_1 - s_2) \exp(s_2 T) \left[-S_0 - \frac{\alpha - \gamma}{\delta} + \left(\frac{\gamma - b + \sigma + \tau_0 \exp(\rho T)}{\delta} + \frac{\alpha - \gamma}{\delta}\right) \exp(-s_1 T) \right] + \frac{\rho \tau_0}{\delta} \exp(\rho T) [s_2 \exp((s_2 - s_1)T) - s_1].$$

This expression is negative, because $s_1 > 0$, $s_2 < 0$ and $S(T) < S_0$. We can thus use the implicit function theorem to

obtain $T = T^* \left(S_0^+, b - \sigma^+, \tau_0^-, \alpha^+, \gamma^+, \delta, \rho, \beta \right)$ in case $S(T) > 0$. If we have $S(T) = 0$, equation (11') gives

$$\frac{\alpha - b + \sigma}{\beta} = \frac{\left(S_0 + \frac{\alpha - \gamma}{\delta}\right)(s_1 - s_2) \exp(s_2 T) + \left(\frac{\alpha - \gamma}{\delta}\right)[s_2 \exp((s_2 - s_1)T) - s_1]}{1 - \exp((s_2 - s_1)T)},$$

which yields $T = T^* \left(S_0^+, b - \sigma^+, \tau_0^0, \alpha^-, \gamma^-, \delta, \rho, \beta \right)$. Q.E.D.

We can distinguish again three regimes: full exhaustion of non-renewables if the backstop is relatively expensive compared to initial marginal extraction costs of non-renewables (i.e., $b - \sigma > \gamma + \tau_0$); partial exhaustion of non-renewables if the backstop becomes cheap enough as oil and gas extraction becomes more expensive ($\gamma - \delta S_0 < b - \sigma - \tau_0 < \gamma$) in which case the terminal stock of non-renewables follows from $G(S(T)) = b - \sigma - \tau(T)$, that is $S(T) = [\gamma - b + \sigma + \tau(T)]/\delta > 0$; and immediate introduction of the backstop and zero oil/gas exploitation if the backstop is always cheaper than non-renewable reserves (i.e., $b - \sigma - \tau_0 < \gamma - \delta S_0$).

In the first regime, $S(T) = 0$ and a backstop subsidy reduces T . This implies that the stock of non-renewables is depleted earlier. Also, a larger initial stock of non-renewables will postpone exhaustion. A permanent boost to fuel demand (higher α) induces faster depletion of non-renewables and brings forward exhaustion. A higher unit cost of non-renewable resource extraction (higher γ) pushes up prices, reduces demand and thus slows down extraction, hence the switch to the backstop occurs later.¹¹ Interestingly, if

¹¹ The effects of s_1 and s_2 and thus of ρ , δ and β on T are more intricate. The effects of extraction costs rising more rapidly as reserves diminish (higher δ) on T are ambiguous; the effect through s_1 and s_2 on T would then be negative while the effect through Γ on T is positive. However, if a more impatient society (higher ρ) leads to more rapid depletion of non-renewables (lower T), it must be the case that with a lower price sensitivity of fuel demand (higher β) it is optimal to have lower fuel prices and thus slower extraction of non-renewables. With $\delta = 0$ and positive extraction, $\alpha - \beta q(t) - \gamma = \lambda(t)$. Using $q(T) = \bar{x}$, we get

$$\rho S_0 = \frac{\alpha - \gamma}{\beta} (\rho T - 1 + e^{-\rho T}) + \bar{x} (1 - e^{-\rho T}) \quad \text{so} \quad \left\{ \frac{\alpha - \gamma}{\beta} (1 - e^{-\rho T}) + \bar{x} e^{-\rho T} \right\} dT = \left\{ \frac{\alpha - \gamma}{\beta} - \bar{x} \right\} \left\{ -1 + e^{-\rho T} + T e^{-\rho T} \right\} d\rho.$$

non-renewables are fully exhausted, an exponential CO2 tax ramp does not affect the time of transition to the backstop and does not affect the oil/gas depletion trajectories. However, in the second regime with $S(T) > 0$, such a CO2 tax ramp encourages society to keep more oil/gas reserves in situ and to switch to non-renewables more quickly. Subsidizing the backstop or increasing oil/gas extraction costs in this regime also induces a bigger final in-situ stock of non-renewable resource reserves (higher $S(T)$) and a more rapid transition to the backstop. It also leads to a higher rate of non-renewable resource extraction at the time that the economy switches to the backstop (i.e., $q(T) = F(b - \sigma)$ goes up if σ or rises). Hence, subsidizing the backstop leads to less oil/gas extraction and, given that the final extraction rate is higher at the date of the switch to the backstop, the initial non-renewable resource extraction rate must be lower.

3.1. Sustaining the first-best outcome

The socially optimal CO2 tax is not an exponentially rising tax.

Proposition 4: The social optimum is sustained in a market economy by a CO2 tax ramp given by $\dot{\tau} / \tau = \rho - D'(E) / \tau < \rho$ and $\sigma = 0$.

Proof: Comparing the optimality conditions for the market economy, (3a')-(3d'), with those of the social optimum, (3a)-(3d), and using $U'(q+x) = p$, we see that to replicate the social optimum the CO2 tax at time t must equal $\pi(t) = \eta(t) - \alpha(t)$ and $\sigma = 0$ with revenue rebated in lump-sum fashion. Using $\omega = \eta - \tau$ in (3a'), we get $\dot{\eta} - \dot{\tau} = \rho(\eta - \tau) + G'(S)q$. Substituting (3a), we obtain $\dot{\tau} / \tau = \rho - D'(E) / \tau < \rho$. Q.E.D.

The rate of change in the optimal carbon tax thus consists of a Hotelling term equal to the rate of time preference minus a term depending on marginal global warming damages.¹² It is thus socially optimal to have the CO2 tax rate growing at a slower rate than the discount rate. Proposition 2 establishes whether a cheaper backstop (e.g., due to technical progress or a subsidy) lowers or increases green welfare. Note that technical progress always boosts social welfare whereas a backstop subsidy financed by lump-sum taxes always lowers social welfare if the socially optimal CO2 tax is in place.

3.2. Green Paradox and second-best outcome in absence of a carbon tax

Sinn (2008), however, argues that a rapidly rising CO2 tax may be tough to sell to the people. Instead, governments may resort to the second-best policy of subsidizing the backstop and financing this with lump-sum taxes whilst ruling out a CO2 tax. Firms then perceive marginal CO2 damages as zero.

Given $\lambda > 0$, a higher ρ decreases T . Whether this also holds for $\delta > 0$ is unclear, since δ appears in s_1 and s_2 as well.

¹² Similar results have been obtained earlier (e.g., Hoel and Kverndokk, 1996).

Therefore, the *FMD* line in figure 2 coincides with the horizontal axis and the resource will be depleted. However, in figure 2 it has been assumed that $b > \gamma$. If the inequality would hold the other way around, the final resource stock is smaller in the competitive economy than in the social optimum. We now consider the effects of levying a constant tax or a constant subsidy on the backstop in the decentralized economy.

Proposition 5: Assume a CO2 tax is not feasible. If the backstop is always more expensive than oil/gas (i.e., $b > \gamma$) and the stock of CO2 is severely damaging (large κ), introducing a tax on the backstop enhances welfare. If the backstop is at some point cheaper than oil or gas ($b < \gamma$) and the stock of CO2 is severely damaging (large κ), subsidizing the backstop enhances welfare.

Proof: We fix all parameters except b and κ . In the competitive economy the time paths of q , S , x , E and T are only determined by b , not by κ and we henceforth parameterize these variables with respect to b . We decompose social welfare in the competitive economy into three parts: private component of social welfare,

$$V(b) \equiv \int_0^{\infty} \exp(-\rho t) [U(q(t; b) + x(t; b)) - G(S(t; b))q(t; b) - bx(t; b)] dt, \text{ green welfare, defined}$$

as $-\kappa\Lambda(b) = -\int_0^{\infty} \exp(-\rho t) \kappa(E_0 + S_0 - S(t; b))^2 dt$ and subsidies. Let us take $\gamma > b > b'$. Suppose policy consists in giving a constant backstop subsidy σ so that the perceived cost becomes b' . Hence, $\sigma = b - b'$. Total discounted cost

of the subsidy is $R(b - b') \equiv \int_{T(b')}^{\infty} \exp(-\rho t) (b - b') \frac{\alpha - b}{\beta} dt = (b - b') \frac{\alpha - b}{\rho\beta} \exp[-\rho T(b')]$. The welfare difference is

$$\Delta(b, b', \kappa) = V(b') + \kappa\Lambda(b') - R(b - b') - [V(b) + \kappa\Lambda(b)] = V(b') - V(b) - (b - b') \frac{\alpha - b}{\rho\beta} \exp[-\rho T(b')] + \kappa[\Lambda(b') - \Lambda(b)].$$

Clearly, $\Delta(b, b', 0) < 0$ because with no negative externalities the decentralized economy is socially optimal. We also observe that, for any given b and b' , $\Delta(b, b', \kappa)$ is monotonic in κ . In the case at hand, $\Lambda(b') - \Lambda(b) > 0$ because a lower backstop price will slow down extraction and leave more oil/gas unextracted. Hence, there exists a critical $\hat{\kappa}(b, b')$, depending on b and b' , such that for $\kappa < \hat{\kappa}(b, b')$ no subsidy should be given and for $\kappa > \hat{\kappa}(b, b')$ a subsidy enhances welfare. For the case $b > \gamma$, the proof is analogous: it is better now to increase the perceived backstop cost because that will lower extraction rates. Q.E.D.

Two remarks are in order. First, once non-renewables are exhausted, it becomes socially optimal to abolish the tax on the backstop. This may lead to a credibility problem. Second, in case of an expensive backstop, an alternative policy is to subsidize the backstop to such an extent that it becomes cheaper than non-renewables. Then the policy is non-marginal, which might work for a very negative externality.

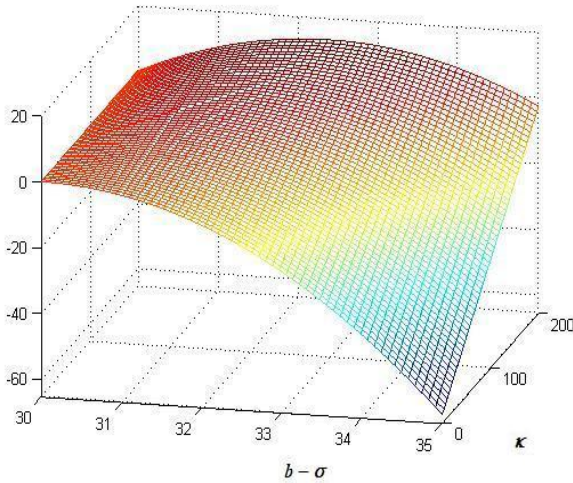
We illustrate our model with an example where $\alpha = 100$, $\beta = 1$, $\gamma = 30$, $b = 30$, $\delta = 0.5$, $\rho = 0.2$, $S_0 = 9$, and $E_0 = 1$. Figure 3 plots the effects of introducing a backstop subsidy and a backstop tax of varying

orders of magnitude on welfare (net of the lump-sum taxes needed to finance the subsidy or the lump-sum transfers made possible by the tax on the backstop)¹³ for different values of the damage parameter κ .

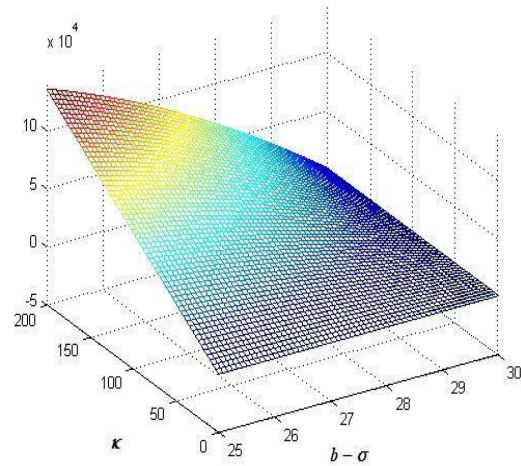
In panel (a) we introduce a tax on the backstop, leading to the backstop cost being larger than γ . In the competitive economy non-renewables will then always be exhausted. The higher the tax and the cost of the backstop, the smaller is the initial rate of extraction and the later exhaustion of oil/gas reserves will take place. This curbs emissions and implies an initial positive effect on green welfare. However, the private component of social welfare falls. With no or little concern about global warming (small κ), taxing the backstop always harms total welfare (see proposition 2 with $\kappa \equiv 0$). Only if society cares a lot about global warming damages and the tax is not too high, is the welfare effect positive. Hence, if the increase in green welfare is large enough, it outweighs the fall in the private component of social welfare. However, too large backstop taxes lower social welfare even if society cares a lot about CO2 damages (high κ). For $\kappa = 200$, welfare is maximized if the backstop tax equals 2.

Figure 3: Backstop subsidies can boost second-best social welfare

(a) Taxing the backstop



(b) Subsidizing the backstop



Panel (b) deals with the case of a subsidy leading to lower backstop costs and therefore to partial oil/gas exhaustion leading to a positive final stock of non-renewables. With no concern about global warming ($\kappa = 0$) introducing a backstop subsidy affects social welfare negatively, since the competitive outcome is socially efficient. However, it turns out that for these parameter values, the net effect of introducing a backstop subsidy is rather small. We also see that even for relatively little concern about global warming,

¹³ Since proposition 3 gives use of the backstop as $x(t) = (\alpha - b + \sigma)/\beta > 0$, $\forall t \geq T$, we have to subtract $\exp(-\rho T)\sigma(\alpha - b + \sigma) / (\beta\rho)$ from social welfare at time zero.

there is a substantial¹⁴ welfare gain from introducing the backstop subsidy. This suggests that with $b > \gamma$ it is better from a welfare perspective to subsidize the backstop so that the effective cost is reduced below 30, rather than taxing the backstop.

To fully realize the first best, the government must implement a CO2 tax ramp, not a subsidy on using a clean backstop. If such an optimal CO2 tax is infeasible, subsidizing the backstop runs into the Green Paradox if the backstop is initially relative expensive. Green welfare will fall as oil/gas reserves are more quickly exhausted, but overall welfare may increase. However, if the backstop is (made) cheap enough compared with current extraction costs of oil and gas, it is optimal to keep some oil/gas reserves unexploited which benefits the environment. Subsidizing the renewable backstop then means that the switch away from oil and gas occurs more rapidly; and also that a bigger fraction of oil/gas reserves remains in situ. CO2 emissions are less, so that the Green Paradox is avoided. An alternative is to compensate the owner of non-renewable resources for keeping some of its reserves unexploited. Interestingly, Ecuador recently demanded \$4.5 billion as compensation to keep oil in the soil and thus preserve the Amazon rain forest and curb CO2 emissions by 410 million tons. In practice, mining companies also attempt to bribe indigenous people to accept their resources being plundered.

4. Extension: Switching from oil/gas to a dirty backstop

In practice, one may have to switch to dirtier energy sources. For example, it takes a lot of effort to recover oil from the Canadian tar sands and this goes hand in hand with high CO2 emissions ($b > \gamma - \delta S$, $\psi > 1$). Alternatively, one may have to switch to coal which is in abundant supply, cheaper than oil or gas, but also leads to more CO2 emissions than conventional oil or gas ($b < \gamma - \delta S$, $\psi > 1$). We obtain the first-best outcomes for such backstops by maximizing social welfare (1') subject to the depletion equation (2) and the equations describing the concentration of CO2 in the atmosphere $\dot{E}(t) = q(t) + \psi x(t)$,

$E(t) = E_0 + S_0 - S(t) + Y(t)$ and $Y(t) \equiv \psi \int_0^t x(s) ds$. Necessary conditions for an optimum are given in the appendix. We suppose $\psi > 1$ and focus in proposition 6 below on the conditions under which it is optimal to use initially only non-renewables and subsequently only the backstop. Proposition 7 then indicates that we cannot exclude simultaneous use of non-renewables and the dirty backstop.

Taking due account of global warming externalities, lemma 1 in the appendix indicates that we should cut back use of the backstop, $x(t) < F(b)$, $\forall t \geq T$. The solution is characterized by $x(t)$ jumping up at time T

¹⁴ Note that in panel (b) the vertical axis is several orders of magnitude larger than in panel (a).

and then gradually falling to zero at the speed $|\lambda_2|$ while $Y(t)$ starts from zero at time T and then asymptotically reaches its steady-state value Y^* . Due to discounting and convex global warming damages, use of the backstop gradually diminishes as time proceeds and vanishes asymptotically. The rate at which use of the backstop falls increases with the rate at which marginal global warming damages increase (κ), the emission intensity of the backstop (ψ), and the sensitivity of demand for fossil fuels with respect to the price ($1/\beta$). Total use of the backstop (Y^*) is less if past use of fossil fuels has already led to a high concentration of CO2 emissions in the atmosphere and if the unit cost of the backstop (b) is high, but more if autonomous demand for energy (α) is high.

Proposition 6: If $\exists T$ such that $x(t) = 0$, $q(t) > 0$, $\forall t < T$, and $q(t) = 0$, $x(t) > 0$, $\forall t > T$ and

$\alpha > b + \psi\kappa(E_0 + S_0)/\rho$, then $S(t)$, $t \leq T$, is given by (8) with s_1 , s_2 and Γ defined in proposition 1 and $S(T) = 0$. The time of the switch from conventional oil and gas to the backstop is given by

$$T = T^* \left(S_0^+, 0, b^+, \alpha^+, \gamma^+, \delta^+, \rho^+, \beta^+, E_0^+, \kappa^+, \psi^+ \right).$$

Proof: For the case at hand the motion of the non-renewable resource is given by (7), resulting in the solution given by (8). Moreover, it was shown in the previous lemma that $S(T) = 0$. Continuity of the Hamiltonian function requires $q(T) = -\dot{S}(T) = x(T)$. Hence, using lemma 1 of the appendix:

$$x(T) = \left(\frac{\rho}{\lambda_1} \right) \left(\frac{\alpha - b}{\beta} \right) - \left(\frac{\psi\kappa}{\beta\lambda_1} \right) [E_0 + S_0] = \frac{(S_0 + \Gamma)(s_1 - s_2) \exp(s_2 T) + \Gamma [s_2 \exp((s_2 - s_1)T) - s_1]}{1 - \exp((s_2 - s_1)T)} \equiv \Phi \left(T, S_0^+, \Gamma^+, s_1^+, s_2^+ \right).$$

We know from proposition 1 that $\Phi_T < 0$, $\Phi_{s_1} > 0$ and $\Phi_{s_2} > 0$. We can solve for T , where we have to take account of the fact that $x(T)$ is a less than proportional function of $(\alpha - b)/\beta$ (i.e., the value of $x(T)$ that would prevail if $\psi = 0$) and a negative function of $(E_0 + S_0)$. It follows that the signs of the partial derivatives of $T^*(.)$ with respect to α , b , E_0 , S_0 , and S_T are the same as those of $T(.)$ as given by equation (9). Note that since $x(T)$ is a negative function of ψ , $T^*(.)$ must be a positive function of ψ . Lemma 3 of the appendix establishes that $S(T) = 0$. Q.E.D.

Novel is the result that if the backstop emits more CO2 per unit of energy (higher ψ) and no simultaneous use of oil/gas and the backstop takes place, oil/gas reserves will be fully exhausted. It then takes longer to exhaust conventional oil/gas reserves in order to postpone use of the dirty backstop. Furthermore, once oil/gas revenues are fully exhausted, use of the backstop is less.

However, simultaneous use of non-renewables and the backstop cannot be excluded.

Proposition 7: If there is simultaneous use of non-renewables and the backstop forever from the outset,

then $S(t) = K \exp(s_2 t) + \frac{(1 - \psi)\rho(\alpha - \gamma) + \kappa(\psi - 1)E_0 + \delta\rho S_0}{\rho\delta\psi}$ with $s_2 < 0$ and

$K = S_0 - \frac{(1-\psi)(\alpha-\gamma)+\gamma-b}{\delta\psi}$. Necessary conditions for this to occur are $1 < \psi < 1 + \rho\delta/\kappa$,

$b + (\psi-1)(\alpha-\gamma) < \gamma$ and $b + (\psi-1)(\alpha-\gamma + \delta S_0) > \gamma - \delta S_0$.

Proof: With $E(t) = E_0 + S_0 - S(t) + Y(t)$ and $Y(t) = \int_0^t \psi x(s) ds$, the necessary conditions give

$\left[\delta - \left(\frac{\psi-1}{\rho} \right) \kappa \right] \dot{S} = - \left(\frac{\psi-1}{\rho} \right) \kappa \dot{Y}$. As $Y(0) = 0$, we have $\left[\delta - \left(\frac{\psi-1}{\rho} \right) \kappa \right] (S(t) - S_0) = - \left(\frac{\psi-1}{\rho} \right) \kappa Y(t)$ which establishes the

first necessary condition. We have $Y(t) = \zeta(S(t) - S_0)$ with $\zeta \equiv \left[\left(\frac{\psi-1}{\rho} \right) \kappa - \delta \right] / \left(\frac{\psi-1}{\rho} \right) \kappa$. So, $\dot{Y} = \zeta \dot{S}$, $\ddot{Y} = \zeta \ddot{S}$. The ordinary differential equation applying to this case reads:

$$-\beta(\dot{q} + \dot{x}) + \delta \dot{S} = \dot{\lambda} + \dot{\mu} = \rho\lambda - \delta q + \rho\mu - \kappa(E_0 + S_0 - S + Y) = \rho(\alpha - \beta(q + x) - \gamma + \delta S) - \delta q - \kappa(E_0 + S_0 - S + Y)$$

or $-\beta \left(-\ddot{S} + \frac{1}{\psi} \ddot{Y} \right) = \rho \left[\alpha - \beta \left(-\dot{S} + \frac{1}{\psi} \dot{Y} \right) - \gamma + \delta S \right] - \kappa(E_0 + S_0 - S + Y)$ and thus

$-\beta \left(-\ddot{S} + \frac{1}{\psi} \zeta \ddot{S} \right) = \rho \left[\alpha - \beta \left(-\dot{S} + \frac{1}{\psi} \zeta \dot{S} \right) - \gamma + \delta S \right] - \kappa[E_0 + S_0 - S + \zeta(S - S_0)]$. The resulting differential equation,

$$\beta \ddot{S} - \rho \beta \dot{S} - \frac{\psi(\rho\delta + \kappa(1-\zeta))}{\psi-\zeta} S = \frac{\psi(\rho(\alpha-\gamma) - \kappa[E_0 + S_0(1-\zeta)])}{\psi-\zeta}, \text{ yields } S(t) = K \exp(s_2 t) + \frac{(1-\psi)(\alpha-\gamma) + \gamma - b}{\delta\psi} \text{ with } K \text{ to be}$$

determined by the initial condition, s_2 the negative root and where we have used that since there is simultaneous use of renewables and non-renewables from the beginning, we have $\rho(b - \gamma) + \kappa(\psi - 1)E_0 + \delta\rho S_0 = 0$. Hence, we need the second necessary condition and $K = S_0 - S(\infty) > 0$, requiring the third necessary condition. Q.E.D.

A moderately dirty and cheap backstop such as coal might satisfy the first condition of proposition 7, that is $1 < \psi < 1 + \rho\delta/\kappa$. The second condition of proposition 7 requires that the backstop is relatively cheap once oil/gas reserves are fully exhausted. The third condition demands that the backstop is relatively expensive at the beginning of the planning period, which requires that costs of oil/gas extraction rise rapidly as reserves are depleted (high δ).¹⁵ If these three conditions are satisfied, proposition 7 states that it is optimal to follow up a period of solely oil/gas depletion with a regime of oil/gas and coal use before moving to a regime of only coal use. Oil/gas reserves will then be fully exhausted if the backstop is relatively expensive compared with the maximum price of fuel and if the backstop is relatively dirty.

To ensure feasibility of proposed optimal programs, we assume $\alpha > b + \psi\kappa(E_0 + S_0)/\rho$.¹⁶ Before we can fully characterize all cases of interest, we define what we mean by an *expensive* or *cheap* backstop.

¹⁵ The third condition, $\rho(b - \gamma) + \kappa(\psi - 1)E_0 + \delta\rho S_0 = 0$, only holds by fluke. But with another starting point, say T , the condition is $b - \gamma + \delta S(T) = (1 - \psi)\kappa E(T)/\rho$, which may be satisfied after an initial interval of time.

¹⁶ Alternatively, $\alpha < b + \psi\kappa E_0/\rho$ so the backstop will never be used or $\alpha = b + \psi\kappa(\tilde{E})/\rho$ for some $\tilde{E} < E_0 + S_0$.

Imagine oil/gas reserves are fully exhausted and no use of the backstop has been made yet, so the CO2 concentration equals $E_0 + S_0$. The marginal cost of using the backstop then equals direct cost plus discounted CO2 damages, $b + \psi\kappa(E_0 + S_0) / \rho$. The marginal cost of using oil/gas equals the extraction cost γ (as $S = 0$) plus discounted CO2 damages $\kappa(E_0 + S_0) / \rho$. Our definition of an *expensive* backstop is then that *after full exhaustion of oil/gas reserves* one would rather have an extra unit of oil/gas than an extra unit of the backstop, that is $b + \psi\kappa(E_0 + S_0) / \rho > \gamma + \kappa(E_0 + S_0) / \rho$. If the backstop is expensive according to this definition, the backstop can still be cheap initially. To see this, rewrite the condition as $b - \gamma + \delta S(t) > (1 - \psi)\kappa[E_0 + S_0 - S(t)] / \rho + (\delta + (1 - \psi)\kappa / \rho)S(t)$. Hence, if the costs of oil/gas extraction do not fall too rapidly with depletion, i.e., $\delta < (\psi - 1)\kappa / \rho$, $b - \gamma + \delta S_0 < (1 - \psi)\kappa E_0 / \rho$ may hold. At the outset the backstop is then cheaper as a consequence of the low δ , meaning that cost of oil/gas extraction does not fall too rapidly with depletion. We use this definition of an expensive and a cheap backstop together with the definition of a moderately and a very dirty backstop to arrive at the four cases sketched in table 2.

Table 2: Different trajectories of non-renewables and backstop

Backstop	Dirty $1 < \psi < 1 + \rho\delta / \kappa$	Very dirty $\psi > 1 + \rho\delta / \kappa$
Expensive $b > \gamma + (1 - \psi)\kappa(E_0 + S_0) / \rho$	1. Oil/gas followed by backstop	2. Like case 1 with possibly the backstop initially
Cheap $b < \gamma + (1 - \psi)\kappa(E_0 + S_0) / \rho$	3. Like case 4 with possibly first oil/gas followed by oil/gas & backstop before only the backstop	4. Only use the backstop

Case 1 corresponds to an expensive and moderately dirty backstop and implies for all $S(t) > 0$ that $b > \gamma - \delta S(t) + (1 - \psi)\kappa[E_0 + S_0 - S(t)] / \rho$. The backstop is unambiguously more expensive than non-renewables, inclusive of CO2 damages, even at the outset (due to, say, a relatively large value of δ), but it is still optimal to use the backstop after exhaustion. The optimum can be derived as follows. Initially there will not be simultaneous use, because that would require $b = \gamma - \delta S(t) + (1 - \psi)\kappa[E_0 + S_0 - S(t)] / \rho$ (see the derivation in proposition 7). If we would start with the backstop, then, according to lemma 2 in the appendix, no transition can take place to using only non-renewables or to simultaneous use. Hence, the optimum is to exhaust non-renewables for an initial period and thereafter use the backstop forever.

Case 2 has an expensive and very dirty backstop with not too rapidly rising costs of oil/gas extraction (δ small). We may then have for some $S(t) > 0$, $b - \gamma + \delta S(t) < (1 - \psi)\kappa[E_0 + S_0 - S(t)] / \rho$, in particular for $S(t) = S_0$. Hence, the backstop might be cheaper, inclusive of CO2 damages, as non-renewables are abundant. If this is not the case, we are back in case 1. So, suppose it holds. Note first that simultaneous use is ruled in view of the first condition in proposition 7. An optimal program would then have the backstop initially. At some instant of time non-renewables take over until they are fully exhausted. Then there is a final interval of time starting at, say \tilde{T} with only the backstop, provided our assumption holds with $E_0 + S_0 - S(T)$ replaced by $E(\tilde{T})$.

Case 4 applies to a cheap and very dirty backstop with slowly increasing costs of oil/gas extraction (δ small). It has for all $S(t) > 0$ that $b < \gamma - \delta S(t) + (1 - \psi)\kappa[E_0 + S_0 - S(t)] / \rho$, hence proposition 7 says that there will never be simultaneous use. Moreover, lemma 2 of the appendix indicates that a transition from non-renewables to the backstop never takes place. Using only non-renewables is also suboptimal, since after exhaustion the backstop is profitable. It is thus optimal to start with the backstop and never use non-renewables as the backstop is already cheap at the outset, $b < \gamma - \delta S_0 + (1 - \psi)\kappa E_0 / \rho$.

Case 3 corresponds to a cheap and moderately dirty backstop with relatively rapid rising costs of oil/gas extraction (δ large). For a large enough initial resource stock it can be that $b > \gamma - \delta S_0 + (1 - \psi)\kappa E_0 / \rho$ so that it is optimal to start with non-renewables. At some instant of time we have that $b = \gamma - \delta S(t) + (1 - \psi)\kappa[E_0 + S_0 - S(t)] / \rho$ and from there on there will be simultaneous use, either forever, or for a finite interval of time along which the non-renewables are exhausted. Thereafter, if accumulated CO2 is not too large, the backstop takes over forever.

In all cases an exogenous fall in cost of the backstop (e.g., due to technical progress) boosts social welfare. And if an optimal carbon tax can be imposed, no backstop subsidy is needed in a decentralized economy. However, things become different if a carbon tax is infeasible and the backstop is subsidized. The results can be derived in a similar way as in proposition 5. The decentralized economy produces the first-best in case of no externalities. Then subsidies or taxes make no sense. With severe externalities, the time path followed by the economy does not depend on κ . Hence, by taking this parameter as a pivotal parameter we can determine the desirability of a tax or a subsidy. However, now more regimes are possible and it is less easy to determine the welfare effects of subsidizing or taxing the backstop. Moreover, as in the case of the clean backstop, policy might suffer from dynamic inconsistency.

In case 4 the backstop is really cheap and subsidizing the backstop reduces welfare, because the first best will be realized in a decentralized economy as production of the backstop is cheaper than extraction. Case 3 can be more complicated, unless it yields the same result as case 4.

Now consider case 1 again. Without a CO₂ tax in the decentralized economy, the stock of CO₂ in the atmosphere after T equals $E(t) = E_0 + S_0 - S(T) + \psi(\alpha - b)(t - T) / \beta, t \geq T$. Now suppose $b > \gamma$, so that in the market economy non-renewables are fully exhausted at T . Introducing a subsidy on the backstop, while keeping the backstop more expensive than the non-renewable, has the effect of still fully extracting the stock of non-renewables, the dirty backstop taking over at an earlier instant of time, and thus green welfare falling unambiguously. For a large enough damage parameter κ or a high enough emission intensity of the backstop ψ , the subsidy is detrimental for social welfare (see proposition 5) and it would be better to tax the dirty backstop. Suppose $b < \gamma$, so that the stock of non-renewables will not be fully exhausted. A subsidy on the backstop will leave a bigger part of the stock of non-renewables unexploited, which benefits green welfare. However, the backstop takes over earlier, so there is a trade off. For ψ large enough, green welfare unambiguously falls. The effect is ambiguous if we vary κ , but intuition suggests that for a small rate of time preference the initial benefit will be dominant. The same reasoning can be applied to case 2.

A danger is that economies switch from oil/gas, as reserves run out and become more expensive, to cheap, abundant coal. However, coal without sequestration leads to considerably more CO₂ emissions. The danger is acute as oil/gas reserves are projected to last at most a few decades whereas coal reserves will last for a further two or three hundred years. It thus seems socially optimal to tax coal. More precisely, our analysis suggests that a period of a rising CO₂ tax when oil/gas reserves are depleted will be followed by a period starting out with a much higher CO₂ tax followed by a gradually falling CO₂ tax once the economy has to rely on coal. Unfortunately, in practice many countries subsidize rather than tax coal for social reasons to do with income support. Taxing emissions from coal would not lead to a Green Paradox. In contrast, a relatively high tax on the use of coal deters rational speculators from depleting oil and gas reserves too quickly and helps in the fight against global warming. Taxing coal postpones the period of simultaneous use of oil/gas and coal and also postpones the time that oil/gas reserves are fully exhausted. An even bigger challenge than coal is how to deter Canada exploiting its expensive and very dirty tar sands. This calls for a steeply rising CO₂ tax and a declining CO₂ tax after the switch to tar sands. Another policy measure might be to subsidize Canada to not extract its resource, unfair as this may sound to non-economists.

5. Extension: Monopolistic non-renewable resource owners

So far, we have discussed socially optimal outcomes and outcomes that would prevail in a competitive market economy. In many cases, the owner of the non-renewable resource is a monopolist (e.g., OPEC). It is well known that with monopolies in natural resource markets, limit pricing may occur (e.g., Hoel, 1978). This means that in the presence of a backstop technology with price b and constant marginal extraction costs of the non-renewable resource γ smaller than b , there is an initial phase until some T_1 where the monopolist keeps the market price of oil or gas below the cost of supplying the backstop price, and subsequently a final phase $(T_1, T_2]$ where the backstop price is undercut by an infinitely small margin. The instants of time (T_1, T_2) are determined endogenously by maximizing over the two parts of the trajectory. With stock-dependent extraction costs matters are a bit more complicated. However, limit pricing still occurs. To see this, and to investigate its consequences we consider a monopolist facing a linear inverse demand function $p(t) = \alpha - \beta q(t)$ and having extraction costs $\gamma - \delta S(t)$. The cost of supplying the renewable backstop is b . The monopolist's problem is then

$$(18) \quad \text{Max}_{q, T} \int_0^T \exp(-\rho t) [\alpha - \beta q(t) - \gamma + \delta S(t)] q(t) dt$$

subject to the depletion equation (2) and the inverse demand function $p(t) = \alpha - \beta q(t) \leq b$. Note that the maximization also takes place with respect to the date T at which extraction definitely stops.

Proposition 8: Suppose that the owner of the non-renewable resource is a monopolist who is faced with a renewable backstop fuel over which it has no control. The Green Paradox prevails if the backstop price is relatively high compared to the initial marginal cost of extracting oil or gas ($b > \gamma$). In that case, extraction of the non-renewable resource occurs more slowly than under perfect competition. If the backstop price is relatively low ($b < \gamma$), extraction occurs faster than under perfect competition while a larger stock of oil and gas reserves is left in situ. In that case, the Green Paradox need not necessarily occur.

Proof: The current value Hamiltonian reads $[\alpha - \beta q - \gamma + \delta S]q - \lambda q$. A necessary condition for optimality is

$\alpha - 2\beta q(t) - \gamma + \delta S(t) = \lambda(t)$ as long as $q(t) > 0$ and $\alpha - \beta q(t) < b$, where $\dot{\lambda}(t) = \rho\lambda(t) - \delta q(t)$. Moreover, at the time T when extraction definitely finishes, the Hamiltonian vanishes.

Case $b > \gamma$: Clearly the resource will be exhausted so $S(T) = 0$ and $p(T) = \alpha - \beta q(T) = b$. Moreover, from $H(T) = 0$ we then have $\lambda(T) = b - \gamma > 0$. If the solution would always be interior ($q(t) > 0$ and

$\alpha - \beta q(t) < b$) until exhaustion, meaning no limit pricing, we would have $[\alpha - 2\beta q(t) - \gamma + \delta S(t)]$ arbitrarily close to $[\alpha - 2\beta(\alpha - b) / \beta - \gamma] = 2b - \alpha - \gamma$ for t close enough to T . However, this cannot be equal to $\lambda(T) = b - \gamma$. Therefore, there must be a phase with limit pricing. Hence, there exist $0 < T_1 < T_2$ such that for $0 \leq t \leq T_1$ we have $p(t) < b$,

and for $T_1 \leq t \leq T_2$ we have $p(t) = b$. In the first interval $\alpha - 2\beta q(t) - \gamma + \delta S(t) = \lambda(t)$ and $\dot{\lambda}(t) = \rho\lambda(t) - \delta q(t)$.

This yields the differential equation $2\beta\ddot{S}(t) - 2\rho\beta\dot{S}(t) - \rho\delta S(t) = \rho(\alpha - \gamma)$ with characteristic roots given by

$$s_1 = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 + 2\rho\delta/\beta} > \rho \text{ and } s_2 = \frac{1}{2}\rho - \frac{1}{2}\sqrt{\rho^2 + 2\rho\delta/\beta} < 0, \text{ so the speed of adjustment } -s_2 \text{ is now}$$

smaller than in the competitive outcome. Apart from this, the stock trajectory is as given in proposition 1. A marginal decrease of the backstop price results in a smaller shadow price λ . Indeed a smaller backstop price reduces the constraint set of the monopolist and thereby the shadow price of the non-renewable resource. Consequently, extraction increases both during the first phase as well as in the second phase. Therefore it takes a shorter period of time to exhaust the resource and in this sense the Green Paradox holds. This result a fortiori obtains if limit pricing occurs from the outset.

Case $b < \gamma$: To have an interesting problem, suppose $\gamma - \delta S_0 < b < \gamma$. Otherwise, extraction would never take place. At the time where the monopolist leaves the market (T), the price must equal the backstop price. Hence, $q(T) = (\alpha - b) / \beta$. Moreover, it should not be profitable to extract anymore, $\alpha - \beta(\alpha - b) / \beta - \gamma + \delta S_T = 0$.

Hence, $S_T = (\gamma - b) / \delta$. It follows, as before, that there is a phase with limit pricing. Two countervailing effects are at work. On the one hand, a smaller backstop price increases the remaining stock of oil or gas kept in situ, which runs counter to the Green Paradox. On the other hand, it increases the final extraction rate and thereby all extraction rates during the regime of limit pricing and of the extraction rates before limit pricing starts. This can only happen if the non-renewable resource is taken out of exploitation earlier than before. Hence, initially extraction is excessive, but it lasts much shorter than before. Q.E.D.

It goes beyond the scope of the present paper to assess the green welfare effect, but we conjecture that the effect is ambiguous and depends crucially on the rate of pure time preference. Our results differ sharply from the arguments of Sinn (2008). Either the backstop is always more expensive to supply than oil or gas and the Green Paradox holds, but then rational speculating monopolistic resource owners pump more *slowly* than under perfect competition. Or the backstop is eventually cheaper to supply than oil or gas in which case rational monopolistic resource owners pump oil and gas more *quickly* than under perfect competition, but then subsidizing the backstop leads to bigger stock of oil and gas reserves that are left in the soil and the Green Paradox need not hold.

6. Conclusions

We show that a smaller initial stock of fossil fuel reserves, a positive shock to demand for fossil fuels, and a lower cost of extracting fossil fuels, mean that fossil fuels are more rapidly exhausted in a first-best economy with a clean backstop. We also show that, if the atmosphere has already been polluted with a lot of CO2 emissions, it is socially optimal to postpone depletion of oil and gas in order to combat global warming. When we allow for global warming externalities, we show that Sinn's Green Paradox arises when the backstop is solar or wind energy, which currently are economically less attractive than oil or gas, but more attractive from an environmental point of view as CO2 emissions are insignificant. If, following Sinn, we suppose that a Hotelling ramp for taxes on CO2 emissions is politically infeasible,

then the government might resort to subsidizing solar or wind energy, as is done on a large scale in Germany. In that case, we show that depletion of oil and gas occurs more rapidly and climate change damages increase. This is what Sinn has in mind when he refers to the Green Paradox. We also show that total welfare might decrease. If the concern for the environment is substantial, it would be better to tax the clean backstop in order to postpone exhaustion. However, if a substantial subsidy renders the clean backstop cheaper than fossil fuel, total welfare will be enhanced if the concern for the environment is large enough.

If the backstop, say, nuclear energy, is relatively cheap and low on CO₂ emissions compared to oil and gas, subsidizing nuclear energy will lead to a bigger final in situ stock of oil and gas reserves and to a higher rate of extraction of oil and gas at the time that the economy switches to nuclear energy. Subsidizing the backstop leads to less extraction so that not all oil and gas reserves will be extracted from the earth. Climate damages will now be less and there is no Green Paradox.

If the backstop is relatively dirty and cheap (e.g., coal), there might be a period with simultaneous use of the non-renewable and renewable fuels. If the backstop is very dirty compared to oil or gas (e.g., tar sands), there is no simultaneous use. The optimum policy requires a rising CO₂ tax plus a gradually declining CO₂ tax once the dirty backstop has been introduced.

If the non-renewable resource is owned by a monopolist, there is a potential for limit pricing. The Green Paradox then prevails if the backstop price is relatively high compared to the initial marginal cost of extracting oil or gas. In that case, extraction of the non-renewable resource occurs more slowly than under perfect competition. If the backstop price is relatively low, extraction occurs faster than under perfect competition while a larger stock of oil and gas reserves is left in situ. Interestingly, the Green Paradox need not necessarily occur yet this is the situation that is closest to what Sinn (2008) had in mind.

It may be worthwhile to extend our analysis in the following directions. First, it may be of interest to allow for imperfect substitution in the demand for the non-renewable and the backstop energy source. This may arise from concerns with security of energy supplies, diversification and/or intermittence of backstops such as wind and solar energy and will lead to the simultaneous use of both the non-renewable and the backstop. Second, it is important to investigate what happens if there are various types of backstop available at the same time. If it is possible to rank them, e.g., clean but competitive (nuclear), clean and expensive (wind, solar, advanced nuclear) and dirty and expensive (tar sands), it is best to go for the cleanest and cheapest backstop. However, with dirty and cheap backstops, matters are more complicated especially if we allow for upward-sloping supply schedules of the backstop. Third, given that once non-renewables are exhausted, it becomes attractive to abolish the tax on the backstop, it is of

interest to investigate credibility aspects of optimal climate change policies. Fourth, the analysis could be extended to an international context by analyzing issues of carbon leakage and ways to sustain international cooperation (see Hoel, 2008; Eichner and Pethig, 2009). Fifth, one could investigate the issues we addressed in this paper within the context of a Ramsey model with capital formation and pollution. Finally, one could use the analysis to empirically investigate the various policies that can be used to combat global warming.

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Appendix: Switching from oil/gas to a dirty backstop

Necessary conditions for an optimum are:

$$(A1a) \quad U'(q+x) - G(S) \leq \lambda + \mu, q \geq 0, \text{ c.s.},$$

$$(A1b) \quad U'(q+x) - b \leq \psi\mu, x \geq 0, \text{ c.s.},$$

$$(A1c) \quad \dot{\lambda} = \rho\lambda + G'(S)q,$$

$$(A1d) \quad \dot{\mu} = \rho\mu - D'(E),$$

$$(A1e) \quad \lim_{t \rightarrow \infty} \exp(-\rho t) [\lambda(t)S(t) - \mu(t)E(t)] = 0,$$

where μ is the (negative of the) shadow price of pollution.

Lemma 1: If $\exists T$ such that $x(t)=0, q(t) > 0, \forall t < T$, and $q(t)=0, x(t) > 0, \forall t > T$, then

$$(A2) \quad x(t) = \left(\frac{\psi\kappa}{\beta\lambda_1} \right) Y^* \exp(\lambda_2(t-T)) \text{ and } Y(t) = [1 - \exp(\lambda_2(t-T))] Y^*, \forall t \geq T,$$

where $\lambda_1 = \left[\rho + \sqrt{\rho^2 + 4\psi^2\kappa/\beta} \right] / 2 > \rho > 0$, $\lambda_2 = \left[\rho - \sqrt{\rho^2 + 4\psi^2\kappa/\beta} \right] / 2 < 0$ and

$$Y^* \equiv \left(\frac{\rho}{\psi\kappa} \right) (\alpha - b) - (E_0 + S_0 - S(T)) > 0 \text{ provided } \alpha > b + \psi\kappa(E_0 + S_0 - S(T)) / \rho. \text{ Furthermore,}$$

$$x(T) = \left[\rho(\alpha - b) - \psi\kappa(E_0 + S_0 - S(T)) \right] / \beta\lambda_1 < F(b).$$

Proof: For $t > T$, (A1b) gives $\alpha - \beta x - b = \psi\mu$ and (A1d) gives $\dot{\mu}(t) = \rho\mu(t) - \kappa(E_0 + S_0 - S(T) + Y(t))$. This yields

$$\dot{x} = -\frac{\rho}{\beta}(\alpha - \beta x - b) + \frac{\psi\kappa}{\beta}(E_0 + S_0 - S(T) + Y) \text{ and } \dot{Y} = \psi x. \text{ The system displays saddlepath stability with one positive}$$

characteristic root λ_1 and one negative characteristic root λ_2 . The asymptotic steady states are

$$\lim_{t \rightarrow \infty} x(t) = 0 \text{ and } \lim_{t \rightarrow \infty} Y(t) = Y^* > 0. \text{ Given that } Y(T) = 0, \text{ the solution trajectory } Y(t), \forall t \geq T \text{ is given by (A2). To}$$

ensure $Y^* > 0$, one requires $\alpha > b + \psi\kappa(E_0 + S_0 - S(T))/\rho$. The expression for $x(T)$ is less than $F(b) = (\alpha - b)/\beta$ as $\lambda_1 > \rho$ and the second term is negative. Q.E.D.

Without CO2 emissions ($\psi = 0$), we have $x(T) = F(b) = (\alpha - b)/\beta$ as in section 2.

Lemma 2: If $\exists T$ such that $x(t) = 0$, $q(t) > 0$, for some interval just before T , and $q(t) = 0$, $x(t) > 0$, for some interval right after T , then $b - \gamma + \delta S(T) + (\psi - 1)\kappa E(T)/\rho > 0$.

If $\exists T$ such that $x(t) > 0$, $q(t) = 0$, for some interval just before T , and $q(t) > 0$, $x(t) = 0$, for some interval right after T , then $b + (\psi - 1)\kappa E(T)/\rho < \gamma - \delta S(T)$.

Proof: Consider the transition at time T from $q > 0$ to $x > 0$. Just before T we have $\alpha - \beta(q + x) - \gamma + \delta S = \lambda + \mu$ and $\alpha - \beta(q + x) - b \leq \psi\mu$. Just after T we have $\alpha - \beta(q + x) - \gamma + \delta S < \lambda + \mu$ and $\alpha - \beta(q + x) - b = \psi\mu$. Hence, $\lambda + \mu < \psi\mu + b - \gamma + \delta S$ just before T and $\lambda + \mu > \psi\mu + b - \gamma + \delta S$ right after T . Hence, at T , $\dot{\lambda} + \dot{\mu} > \psi\dot{\mu} + \delta\dot{S}$. Therefore, $\rho(\lambda + \mu) - \psi\rho\mu > (1 - \psi)\kappa(E_0 + S_0 - S(T))$ from the necessary conditions (A1c) and (A1d) and making use of (2). This can be rewritten as $b + (\psi - 1)\kappa E(T)/\rho < \gamma - \delta S(T)$. The proof of the second statement of the lemma is similar. Q.E.D.

Lemma 3: If $\exists T$ such that $x(t) = 0$, $q(t) > 0$, $\forall t < T$, and $q(t) = 0$, $x(t) > 0$, $\forall t > T$, $S(T) = 0$.

Proof: We know from (A1e) that $\exp(-\rho t)[\lambda(t)S(t) - \mu(t)E(t)] \rightarrow 0$ as $t \rightarrow \infty$. It follows from the derivations in lemma 1 that along the program under study $x(t) \rightarrow 0$ as $t \rightarrow \infty$. Hence $E(t)$ approaches a constant and, from (A1b) $\mu(t)$ goes to a constant as well. Moreover, from (A1c) λ is growing at the constant rate ρ from T on. So, the necessary condition reads $\lambda(T)S(T) = 0$. Given the optimal solution trajectory for the use of the backstop and for accumulated CO2 emissions given in lemma 1, we can calculate the shadow price at time T :

$$\lambda(T) = \frac{\psi - 1}{\psi} \left[\left(\frac{\lambda_1 - \rho}{\lambda_1} \right) (\alpha - b) + \frac{\kappa\psi}{\lambda_1} (E_0 + S_0 - S(T)) \right] + b - \gamma + \delta S(T)$$

It follows from the condition $\alpha > b + \psi\kappa(E_0 + S_0 - S(T))/\rho$ of lemma 1 and from

$b - \gamma + \delta S(T) + (\psi - 1)\kappa E(T)/\rho > 0$ of lemma 2 that $\lambda(T) > \frac{\psi - 1}{\psi} \frac{\kappa\psi}{\rho} (E_0 + S_0 - S(T)) + b - \gamma + \delta S(T) \geq 0$, so that $S(T) = 0$. Q.E.D.